Approximate Bifurcation Load of Short Thin-Walled Laminate Plate Structures with Imperfection

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Abstract. The lowest approximate bifurcation load of a real structure with initial imperfection is determined. The investigation is performed for a Z-column made of carbon-epoxy laminate. The amplitude of initial imperfection is small. The column is simply supported on both ends. All calculations are performed by the finite element method and Koiter's method. First, an eigenvalue buckling problem is solved to determine bifurcation loads. Next, post-buckling equilibrium paths for the plate structures are determined to calculate the lowest approximate bifurcation loads by the P-w and P-w^2 methods, the inflection point method and the Koiter's method.

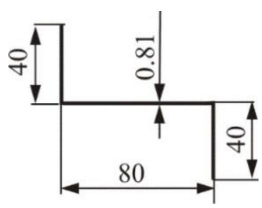
The behavior of a real plate structure under compression can be discussed using a post-buckling equilibrium path. If all types of imperfection can be neglected, then the structure only undergoes shortening in the pre-buckling state and the loss of stability takes place. It is a point of bifurcation, which describes a buckling or bifurcation load. To determine this load, an eigenvalue buckling problem has to be solved. If a plate structure is considered, then the post-buckling equilibrium path is always stable. However, it should be stressed that a significant overload leads to variations in buckling modes. Real plate structures are always prone to inaccuracies, in particular geometric imperfections. The behavior of these structures differs from that observed for perfect structures. In the pre-buckling state, geometric imperfections cause additional bending. Therefore, the bifurcation effect does not exist because there is no unstable equilibrium path. Consequently, the value of buckling load, understood as bifurcation load, cannot be determined. An approximate buckling load should be introduced, equivalent to the buckling load of a real structure. The estimation of the lowest bifurcation load for a real structure can be done by assuming a load corresponding to the point of stiffness change in the post-buckling equilibrium path.

RESULTS

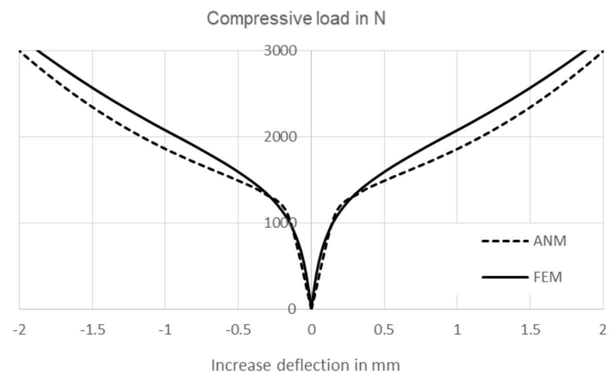
The study was performed on a thin-walled carbon-epoxy laminate Z-column under uniform compression. The dimensions of its cross-section are shown in Fig. 1(a). The laminate had ply orientation $[60/0_2/\text{-}60_2/60_3/\text{-}60_2/0_3/\text{-}60_2/0/60_2]_T$. The length of the specimen was 330 mm. The amplitude of initial imperfection related to the first buckling mode was set equal to 0.1 mm. The carbon-epoxy laminate has the following mechanical properties: Young's modulus in fibre direction (i.e., direction 1) – 170 GPa and in fibre transverse direction (i.e., direction 2) – 7.6 GPa, respectively; Poisson's ratio in plane 1-2 – 0.36; shear modulus in plane 1-2 – 3.52 GPa. The Z-column is simply supported on both ends.

TABLE 1. Lowest bifurcation loads and approximate buckling loads obtained with different methods, in N

Case	Bifurcation loads	***P-w*** method	***P-w***^{2} method	Koiter's method	Inflection point method
FEM	1790	1220	1700	1700	2300
ANM	1640	1370	1480	1480	2070



(a)



(b)

FIGURE 1. Dimensions of the cross-section in mm (a) and post-buckling equilibrium paths of the real Z-column (b)

The eigenvalue problem was solved with the first order approximation of Koiter's method [1] and the Lanczos method by finite element analysis. The bifurcation loads were determined (details in Table 1). Simulations of the post-buckling problem were performed with the second order approximation of Koiter's method [1] and the Newton-Raphson method by finite element analysis. Post-buckling equilibrium paths were obtained by the two methods (Fig. 1(b), where ANM – Koiter's method, FEM – finite element analysis). Next, approximate buckling loads were determined with the following methods: *P-w*, *P-w*^{2}, the inflection point method, and Koiter's method [2-3]. All results are listed in Table 1.

CONCLUSIONS

The Koiter's asymptotic method and the FEM are very useful tools for determining approximate buckling loads for all types of real structures. The lowest approximate loads obtained with the *P-w* and *P-w*^{2} methods and Koiter's method have always been lower than the bifurcation ones. When the amplitude of imperfections is low, the approximate buckling loads are closed to the bifurcation loads. In this case, the inflection point method always tend to overestimate the results. The results obtained with all methods and the bifurcation loads are similar.

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